Enhanced Transmission Line Theory: Frequency-Dependent Line Parameters And Their Insertion in a Classical Transmission Line Equation Solver

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***Abstract*—In this paper, a modified enhanced transmission-line theory taking into account higher-order modes is described. The new equations are based on the correction of the per-unit-length parameters that become frequency dependent and contain the radiation resistance. In particular, the per-unit-length resistance takes into account this radiation resistance and an additional resistance proportional to the imaginary part of the characteristic impedance. This additional resistance is identified as responsible of the current attenuation on the line. These new parameters are described in a per-unit-length RLCG form and can then easily be introduced in a classical transmission-line equation solver. Satisfactory results are obtained when comparing this new model to a full-wave model.**

Keywords—Transmission line; per-unit-length parameters; radiation resistance; automotive electromagnetic compatibility; cable harnesses, electromagnetic interference.

I. Introduction

The electromagnetic compatibility (EMC) characterization of a vehicle requires taking into account several components of its electrical/electronic architecture (EEA) such as the vehicle’s metallic structure, the cable harnesses and the pieces of equipment that are considered as terminal loads. One important point of automotive EMC investigations concerns the electromagnetic field coupling to the harnesses since they play an important role in the propagation of induced currents and voltages to the input/output pins of the equipment. Furthermore, EMC evaluations have to be considered in the early stage of the development of a vehicle project in order to predict the field coupling levels onto wire bundles and determine worst cases. Ideally, this should be accomplished with the resolution of Maxwell's equations for all the conductors in the bundles, and taking into account all the boundary conditions (loads), but this approach is totally unrealistic in the case of an entire vehicle. An alternative solution that is very frequently chosen consists in exploiting the transmission line theory (TLT) and the field-to-transmission line coupling formalism, such as Agrawal's model [1], to deal with these issues. Nevertheless, this theory is limited by assumptions [2] based mainly on the condition that the distance between the line and its return path should be much smaller than the minimum considered wavelength of the incident electromagnetic field. In addition, the classical transmission-line theory is not valid at resonance frequencies even when the condition of distance is satisfied.

In many automotive harness configurations, the classical TLT cannot be used due to the presence of non-TEM modes. Furthermore, the distance of the lines to their return path through the metallic structure of the vehicle is sometimes not sufficiently small compared to the minimum considered wavelength. Therefore, one may suggest a full-wave resolution taking into account all the physical phenomena. However, this requires numerical tools far too time and memory consuming, which makes them simply impossible to use for such complex configurations. Our aim in this work is to extend classical TL equation solvers to this domain where they have often been inaccurate.

Some techniques, modifying the telegrapher's equations in order to be valid where the conditions on the wavelength are no longer respected, have already been developed for several fields of applications [3-7]. But these solutions either need entirely new software development or are based on time consuming iterative methods.

In order to deal with these limitations, a new TL model has been developed. The constitutive equations of this model are directly derived from Maxwell's equations considering only the thin wire approximation [2, 8]. These enhanced equations are then embedded into the classical TL equations’ structure and adapted to the classical TL equation solver.

Hence, the main objective of the new approach proposed in this paper is the insertion of modified enhanced per-unit-length (p.u.l.) parameters in a classical TL equation solver. These new p.u.l. parameters are also frequency-dependent complex values [8] and the imaginary parts are related to the radiated energy.

II. Enhanced p.u.l. Parameters

Let us consider the transmission line geometry given in figure 1. From Maxwell's equations, and using the thin wire

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assumption, the field-to-transmission line coupling equations are given by [2]:

$$\frac{dV^{s}(z)}{dz} + j\omega\frac{\mu_0}{4\pi}\int_0^L g(z-z')I(z')d(z') = E_z^e(h,z) \tag{1}$$

$$\frac{d}{dz}\int_0^L g(z-z')I(z')dz' + j\omega 4\pi\varepsilon_0 V^S(z) = 0 \tag{2}$$

$V^S(z)$ represents the scattered voltage as defined in the classical TLT [1, 2], $I(z')$ is the current amplitude, $E_z^e(h,z)$ is the incident tangential electric field, z and z' stand respectively for the position of the observation point and the source points to the origin, and $g(z-z')$ is the Green's function of this configuration. In order to calculate the above integral, the observation point is considered far enough from the ends of the wire and the current locally constant.

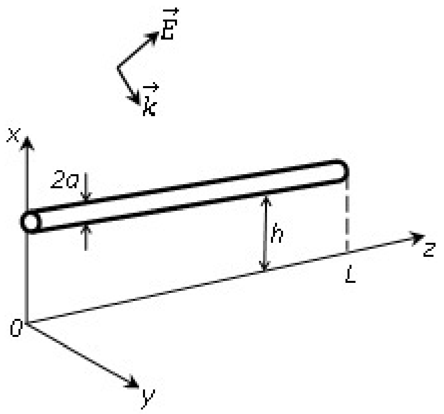


Fig. 1. Geometry of the studied problem.

Thus, considering these assumptions, the integral limits can be taken from $-\infty$ to $+\infty$ and the current distribution placed out the integral [2]. Then, from [8] and using some Bessel functions properties, we can easily find:

$$\int_{-\infty}^{+\infty} g(z-z')dz' = \pi[Y_0(2hk) - Y_0(ak)] + j\pi[J_0(2hk) - J_0(ak)] \tag{3}$$

Here J_0 and Y_0 are, respectively, the 0 order Bessel functions of first and second kind.

Now, using the expansion form of Y_0 the above result of the integral can be rewritten as [9]

$$\int_{-\infty}^{+\infty} g(z-z')dz' = 2\,ln\left(\frac{2h}{a}\right) + C_F \tag{4}$$

Where C_F is the correction factor due to higher-order modes.

In the following, this correction factor to be applied with respect to the classical p.u.l. parameters will be written in a more compact form to split the correction into real part and imaginary part. Therefore, let's note the real part of the correction as:

$$\begin{aligned}\Re(C_F) = {} & +2\left\{[ln(hk)+\gamma][J_0(2hk)-1] - \left[ln\left(\frac{ak}{2}\right)+\gamma\right][J_0(ak)-1]\right\} \\ & -2\left\{\left[\sum_{k=1}^{\infty}(-1)^k\frac{(hk)^{2k}}{(k!)^2}\left(\sum_{m=1}^{k}\frac{1}{m}\right)\right] - \left[\sum_{k=1}^{\infty}(-1)^k\frac{\left(\frac{ak}{2}\right)^{2k}}{(k!)^2}\left(\sum_{m=1}^{k}\frac{1}{m}\right)\right]\right\}\end{aligned} \tag{5}$$

and the imaginary part as:

$$\Im(C_F) = \pi[J_0(2hk) - J_0(ak)] \tag{6}$$

Using these definitions, the enhanced TL equations can now be rewritten as:

$$\frac{dV^S(z)}{dz} + I(z)\left(j\omega L^{HF} + R^{HF}\right) = E_z^e(h,z) \tag{7}$$

$$\frac{dI(z)}{dz} + V^S(z)\left(j\omega C^{HF} + G^{HF}\right) = 0 \tag{8}$$

where

$$L^{HF} = L_0' + \frac{\mu_0}{4\pi}\Re(C_F) \tag{9}$$

$$R^{HF} = -\omega\frac{\mu_0}{4\pi}\Im(C_F) \tag{10}$$

$$C^{HF} = \frac{C_0'\left[2\,ln\left(\frac{2h}{a}\right) + \Re(C_F)\right]}{\dfrac{4\,ln^2\left(\frac{2h}{a}\right) + 4\Re(C_F)\,ln\left(\frac{2h}{a}\right) + \Re(C_F)^2 + \Im(C_F)^2}{2\,ln\left(\frac{2h}{a}\right)}} \tag{11}$$

$$G^{HF} = \frac{\omega C_0'\Im(C_F)}{\dfrac{4\,ln^2\left(\frac{2h}{a}\right) + 4\Re(C_F)\,ln\left(\frac{2h}{a}\right) + \Re(C_F)^2 + \Im(C_F)^2}{2\,ln\left(\frac{2h}{a}\right)}} \tag{12}$$

and where the classical p.u.l. parameters are given by:

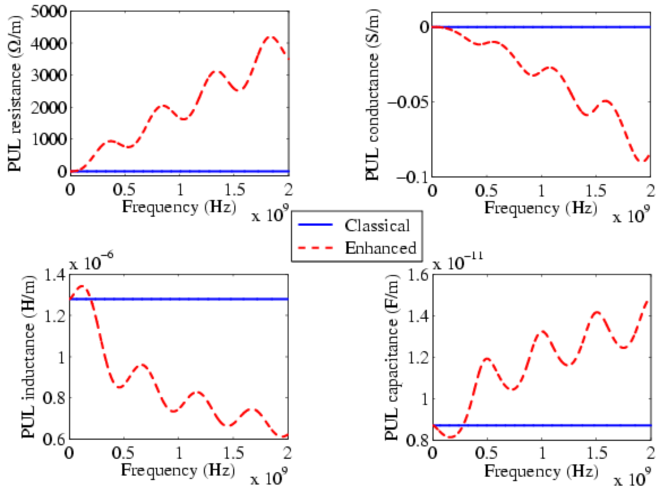
$$L_0' = \frac{\mu_0}{2\pi}ln\left(\frac{2h}{a}\right) \tag{13}$$

and

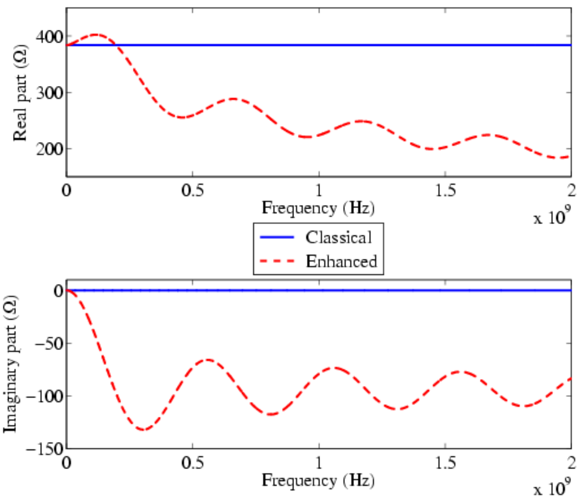
$$C_0' = \frac{2\pi\varepsilon_0}{ln\left(\frac{2h}{a}\right)} \tag{14}$$

It is interesting to note that the above enhanced system of TL equations (7) and (8) has now exactly the same form as the classical one. Therefore, all the known methods of resolution can be used. In our case, we use the Baum, Liu and Tesche (BLT) formulation of the TL equations [10].

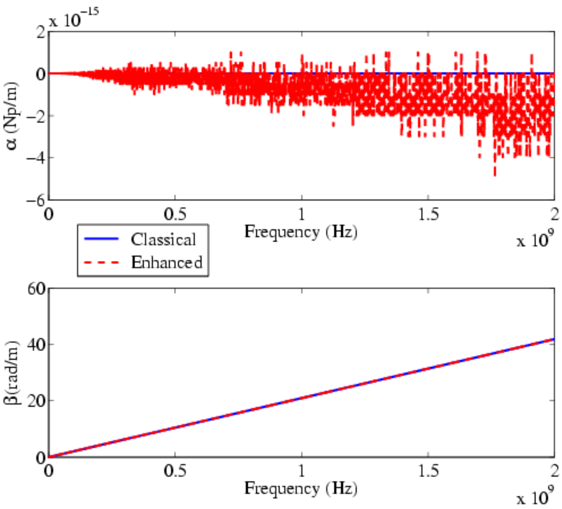
The comparison between the classical and the enhanced TL models in terms of the p.u.l. parameters, characteristic impedance and propagation constant is given in figures 2 for a horizontal TL which dimensions are a=1mm, h=30cm and L=5m. From Fig. 2a, we note that the enhanced p.u.l. parameters differ from the classical ones especially when the height is no longer sufficiently small compared to the wavelength. This remark was already made in [8]. However, in this case, in addition to describing the enhanced p.u.l. parameters as combinations of the classical ones and the correction due to the high frequency effects, the enhanced p.u.l. resistance and conductance are clearly presented as distinct p.u.l. parameters and not only as imaginary parts of the enhanced inductance and capacitance as in [8]. It is to be noted that the new parameter identified as a negative conductance means that energy is created (radiated) by the line (see [8] and some of its references) and the positive p.u.l. resistance is related to energy consumption on the line. However, if these parameters are directly used in a BLT TL equation solver, the results would not be more accurate: this is explained by the continuity equation that relates the lost radiated energy (represented by R^{HF}) and the mechanism of radiation associated with a current source (represented by the negative value of G^{HF}). This is confirmed by the result of the propagation constant in Fig. 2c that is identical to the classical one (the rapid oscillations are due to numerical rounding errors) and denotes that there is no additional attenuation even if the enhanced p.u.l. parameters are used. Besides, we can also observe in Fig. 2b that the characteristic impedance is now complex and frequency-dependent. When the height is much smaller than the wavelength, the quasi-static approximation is valid, the real part is identical to the classical characteristic impedance and the imaginary part is null. However, when the height becomes important in comparison to the wavelength, the real part of the characteristic impedance fluctuates and the imaginary part starts to play a role.



(a) Comparison between p.u.l. parameters.



(b) Comparison between characteristic impedance.



(c) Comparison between propagation constant.

Fig. 2. Comparison between classical and enhanced TL models.

Moreover, it will be shown hereafter that the imaginary part of the characteristic impedance is related to the radiation resistance. We will show that its distribution along the line length will lead to more accurate results.

III. ENHANCED P.U.L. PARAMETERS RESULTS

In order to study the behavior of this new model, some simulations were carried out. The p.u.l. parameters are calculated either using their classical form or using the enhanced version presented previously. Both results are compared to those obtained with an electromagnetic full-wave method of moments (MoM) solver (FEKO). The current in the load is calculated as a function of frequency of the excitation signal.

The configuration studied is presented in Fig. 3. It consists of a perfectly conducting and non-coated wire of 5m length and 1mm radius at 30cm above a perfectly conducting ground plane. It is fed by a 1V sinusoidal voltage source and loaded with a 1Ω resistance.

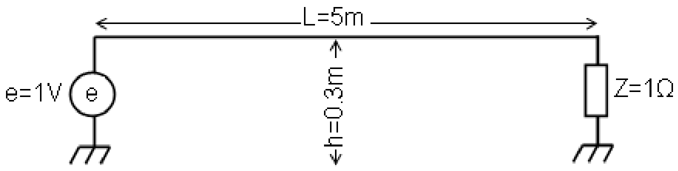


Fig. 3. Setup of the validation circuit.

Figures 4 depict the comparison of current amplitudes as a function of the frequency in the load. When the enhanced p.u.l. parameters are used in a BLT TL equation solver, we notice that the results obtained are not more accurate (Fig. 4b) and do not differ much from the ones obtained by the classical equations (Fig. 4a). This is due to the fact that, as stated previously, although these parameters are enhanced, they only split the energy into a TEM mode (real part of the new characteristic impedance) and a radiated mode (imaginary part of the new characteristic impedance) and therefore, energy onto the line is still stored (the attenuation constant is null).

As observed with a time-domain solver based on Maxwell's equations [2], dissipation only occurs at the extremities of the line and the only way found to dissipate this energy is by calculating more appropriate frequency-dependent reflection coefficients at both ends of the TL. However, this is very difficult to achieve when using a standard TL equation solver. A more convenient solution is the use of an additional resistance representing the radiation losses on the TL.

IV. MODIFIED ENHANCED P.U.L. PARAMETERS

If the enhanced p.u.l. parameters do not lead to better results, this is not due to their inaccuracy but only to their intrinsic properties which splits energy into TEM and radiated modes.

The antenna mode energy is represented by the imaginary part of the characteristic impedance. Therefore, the power to be radiated can be associated with the ratio between the imaginary part and the real part of the characteristic impedance, which is shown to be equivalent to the definition of the quality factor of the TL.

From the definitions of the enhanced parameters (9) to (14) and using the definition of the characteristic impedance as the ratio of the impedance to the admittance:

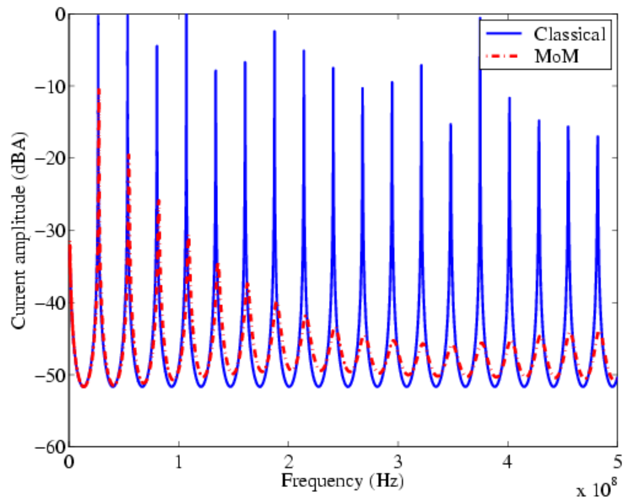
$$Z_c = \sqrt{\frac{R^{HF} + j\omega L^{HF}}{G^{HF} + j\omega C^{HF}}} \qquad (15)$$

and after some mathematical developments, we obtain:

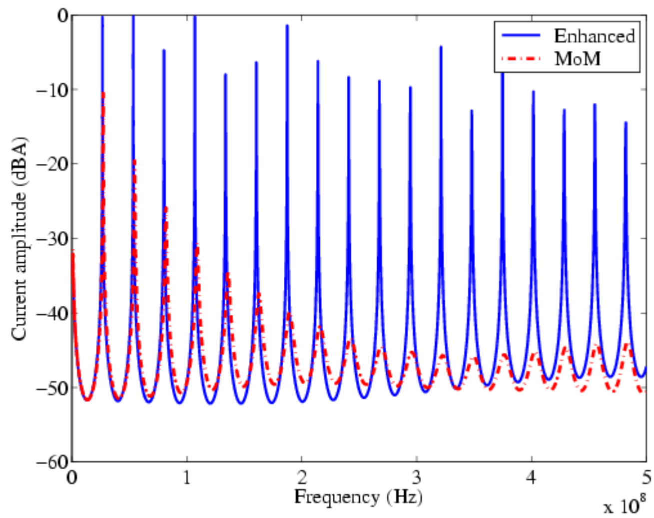
$$Z_c = \sqrt{\frac{L^{HF}}{C^{HF}}} - j\frac{R^{HF}}{k} \qquad (16)$$

which leads to the following formula for the quality factor

$$Q_{line} = \frac{k}{R^{HF}}\sqrt{\frac{L^{HF}}{C^{HF}}} \approx \frac{\omega L^{HF}}{R^{HF}} \qquad (17)$$



(a) Using the classical p.u.l. parameters.



(b) Using the enhanced p.u.l. parameters.

Fig. 4. Current amplitude as a function of the frequency at the end of the line.

An approximation of the term on the right-hand side in (17) is valid if $R^{HF} G^{HF} << L^{HF} C^{HF} \omega^2$ which is always verified in practical situations.

As stated above, using these new p.u.l. parameters does not affect the attenuation constant α, which leads to unchanged results. In order to make this attenuation constant non null, an additional resistance R_+ will be introduced. This additional resistance should be related to the imaginary part of the characteristic impedance, since it represents the antenna mode energy.

If one considers the wave attenuation of the current squared (or the voltage squared) over the length L of the line, it can be written under the form of:

$$A(L) = exp(-2\alpha L) \qquad (18)$$

When an additional resistance R_+ is added to the p.u.l. parameters in series with the radiation resistance R^{HF} and the enhanced p.u.l. inductance L^{HF}, the squared propagation constant becomes:

$$\gamma^2 = \left(R^{HF} + R_+ + j\omega L^{HF}\right)\left(G^{HF} + j\omega C^{HF}\right) \tag{19}$$

Since $\gamma = \alpha + j\beta$, after some mathematical developments, it follows that:

$$\alpha^2 = \frac{(R_+)^2 C^{HF}}{4L^{HF}} \tag{20}$$

This energy dissipation $A(L)$ should be equal to the one produced by the radiation resistance R^{HF}.

Identifying the attenuation along the line $A(L)$ in (18) with the quality factor of the line in (17) yields:

$$1 - exp\left(R_+\sqrt{\frac{C^{HF}}{L^{HF}}}L\right) = \frac{R^{HF}}{\omega L^{HF}} \tag{21}$$

Finally, we obtain:

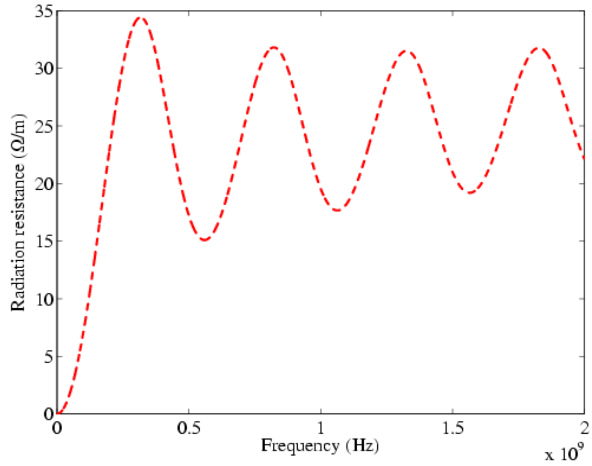
$$R_+ = \frac{1}{L}\sqrt{\frac{L^{HF}}{C^{HF}}}\, ln\left(1 - \frac{R^{HF}}{\omega L^{HF}}\right) \approx \frac{R^{HF}}{kL} \tag{22}$$

As expected, the p.u.l. additional radiation resistance is equivalent to the absolute value of the imaginary part of the characteristic impedance.

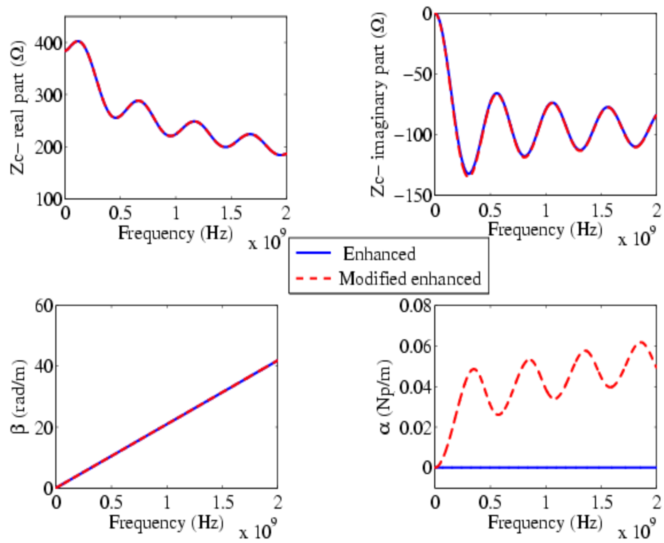
Figures 5 represent the characteristics of the modified enhanced p.u.l. parameters of the previous horizontal wire. It is interesting to note that the modified enhanced p.u.l. parameters, that consist only in a modification of the p.u.l. radiation resistance in such way that these parameters lead to dissipate the energy on the TL, do neither affect the phase constant nor the characteristic impedance but only the attenuation constant (Fig. 5b). This can be explained by the fact that the additional resistance, R_+ (Fig. 5a), is relatively small compared to the p.u.l. impedance.

V. MODIFIED ENHANCED P.U.L. PARAMETERS RESULTS

The same TL configuration was simulated using the modified enhanced p.u.l. parameters and the corresponding results are given in figure 6. We can see, for example (Fig. 6a), that when the classical p.u.l. parameters lead to a current amplitude of 0 dB, the use of the modified enhanced ones predict -10 dB as the full-wave solver does. Generally, the modified enhanced transmission line approach leads to better results, that are comparable to those obtained by a full-wave solver. Below roughly 300 MHz, results obtained by our method are in good agreement with the MoM whereas the classical transmission line exhibits unrealistic resonance magnitudes. Beyond this frequency, the enhanced model does not provide good results. This can be explained by the effect of the vertical wires that are only modeled for the full-wave solver (for practical reasons) and not in the TL approach. This effect can be approximately taken into account through the addition of a p.u.l. resistance for the vertical wires calculated from the radiation resistance of a monopole antenna [11,12]. As expected, this produces slightly better results (5dB) (Fig. 6b).



(a) The additional correction resistance.

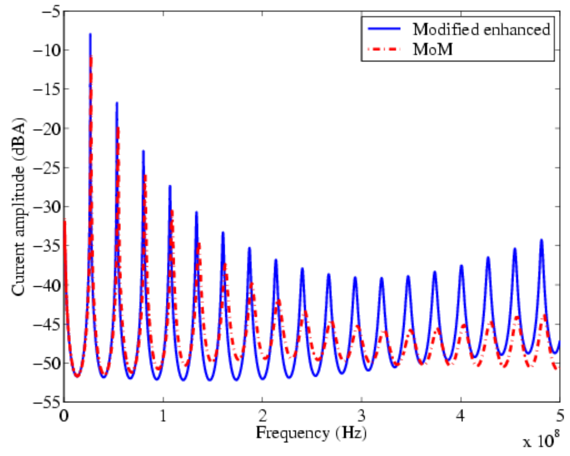


(b) Characteristic impedance and propagation constant.

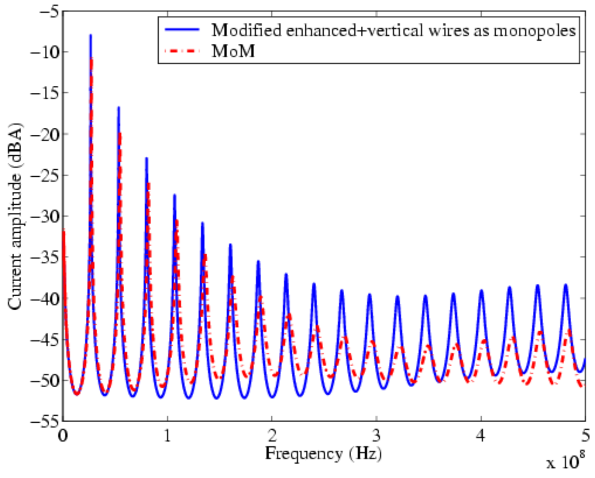
Fig. 5. Characteristics of the modified enhanced p.u.l. parameters.

However, there are still some differences higher in frequency. This is due to the monopole model used for the vertical wires and to the effect of the coupling between the horizontal and vertical wires. Nevertheless, in reality, wires are not perfect conductors and as a consequence, current amplitudes induced in the loads are even lower than predicted by the above results.

Therefore, the proposed model leads to responses much more realistic than the classical TLT model, and provides a better approximation, especially for the most critical first resonances.



(a) Using the modified enhanced p.u.l. parameters.



(b) Using the modified enhanced p.u.l. parameters and including the vertical wires.

Fig. 6. Current amplitude as a function of the frequency at the end of the line.

VI. CONCLUSIONS

A more explicit form of the per-unit-length (p.u.l.) parameters has been presented. In this new model, the enhanced p.u.l. parameters are not only frequency-dependent but also expressed as the sum of the classical parameters and a correction taking into account the high frequency effect. Furthermore, when the height of the line is still smaller than the minimum considered wavelength of the exciting signal, they are easily simplified to become the classical parameters. These new parameters are put under a per-unit-length RLCG form and can then be used in a classical transmission line (TL) solver without any need to develop a new solver.

Another advantage of these new parameters is that the correction is known *a priori* and, therefore, the validity of the approximation done with the classical parameters can be easily quantified.

Nevertheless, the enhanced p.u.l. parameters do not lead to better results although they are different from classical ones. This is explained by the fact that all these parameters (in their first expression) lead only to energy decomposition within the TL.

Thus, they cannot be used in their first expression, but still need a further correction to dissipate energy. This correction is introduced by an additional resistance in series responsible for the attenuation of the antenna mode energy along the line. This additional resistance is related to the radiation resistance or the imaginary part of the characteristic impedance through the quality factor of the line. Its calculation is based on a fully heuristic approach based on the inclusion of losses equivalent to the ratio of power radiated by the transmission line seen as a one-dimensional cavity. The results are comparable to those obtained with a full-wave solver even at resonance, which was the main focus of our study.

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